4.3 #9, #25, #45, #61, #75, #81

Product Formula: $(fg)' = f'g + g'f$

Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$

(9) $S(x) = \frac{2}{x}$ a) Power Rule: $S(x) = 2x^{-1}$

$S'(x) = -2x^{-2} = \boxed{\frac{-2}{x^2}}$ ✓

b) Quotient Rule: $\frac{2}{x}$ f $f = 2$, $g = x$

g $f' = 0$, $g' = 1$

$\frac{f'g - g'f}{g^2} = \frac{0(x) - 1(2)}{(x)^2} = \frac{0-2}{x^2}$

$= \boxed{\frac{-2}{x^2}}$ ✓

(25) $y = (2x^2 - 4x + 1)^2$, so $y' = 2(2x^2 - 4x + 1)' * (4x - 4)$

$(8x - 8)(2x^2 - 4x + 1)$

power plus chain rule

$2x^2 - 4x + 1$

$*\quad 8x - 8$

$16x^3 - 32x^2 + 8x - 16x^2 + 32x - 8$

Combine like terms

$\boxed{16x^3 - 48x^2 + 40x - 8}$ ✓

Without chain rule: $y = (2x^2 - 4x + 1)^2$

$= (2x^2 - 4x + 1)(2x^2 - 4x + 1)$

$= 4x^4 - 8x^3 + 2x^2$

$-8x^3 + 16x^2 - 4x$

$+ 2x^2 - 4x + 1$

$4x^4 - 16x^3 + 20x^2 - 8x + 1$

$\boxed{y' = 16x^3 - 48x^2 + 40x - 8}$ ✓

(45) $y = \dfrac{\left(\frac{1}{x} + \frac{1}{x^2}\right)}{x+x^2}$ Simplify Algebra first

Step 1 Find LCD $= x^2$, so $\dfrac{\left(\frac{x+1}{x^2}\right)}{\left(\frac{x+x^2}{1}\right)} = \dfrac{x+1}{x^2} \cdot \dfrac{1}{x+x^2}$ ($x+x^2 = x(1+x)$)

Rule: $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

Step 2 $= \dfrac{x+1}{x^2} \cdot \dfrac{1}{x(1+x)} = \dfrac{1}{x^3}$

Step 3 $y = x^{-3}$

$y' = -3x^{-4} = \boxed{\dfrac{-3}{x^4}}$

(61) $y = (x+2)\left(\dfrac{x}{x+1}\right)$, so $f = x+2$, $f' = 1$ $\quad g = \dfrac{x}{x+1}$

$g' =$ Quotient Rule

Step 1 Algebraically clean it up!

$(x+2)\left(\dfrac{x}{x+1}\right) = \dfrac{x^2+2x}{x+1}$ (f over g) } Use Quotient Rule b/c divide

Step 2 Find $f = x^2+2x$, $f' = 2x+2$ and $g = x+1$, $g' = 1$

Step 3 Formula & $\dfrac{f'g - g'f}{g^2} = \dfrac{(2x+2)(x+1) - (x^2+2x)}{(x+1)^2}$

Step 4 $= \dfrac{2x^2+2x+2x+2-x^2-2x}{(x+1)^2}$ normally don't FOIL denom.

$= \boxed{\dfrac{x^2+2x+2}{(x+1)^2}}$ } Double check to ensure it does not factor further.

(75) Day 1 = 20 @ $7
Day 2 = 17 @ $8
Dropping @ −3 per day +1 per day

Revenue = P * Q Let (x) = day #

Price = 7 + 1x
Quantity = 20 − 3x
} $R = (7+x)(20-3x)$ Product Rule / FOIL 1st

$= 140 - 21x + 20x - 3x^2$
$R' = -3x^2 - x + 140$
$R' = -6x - 1$
$R'(1) = -6(1) - 1 = -\$7$
$R'(2) = -6(2) - 1 = -\$13$

(81) P(t) − I(t) = How many oil barrels left over after import
and
$\frac{I(t)}{P(t)}$ = what % of oil being sent to US.

P = Production in Mexico
I = Imported by U.S.

$\frac{d}{dt}\left[\frac{I(t)}{P(t)}\right] = \frac{d}{dt}\left[\frac{2.1-0.11t}{3.9-0.10t}\right]$ Quotient Rule b/c division

Formula: $\frac{f'g - g'f}{g^2}$ $f = 2.1 - 0.11t$, $f' = -0.11$ $g = 3.9 - 0.10t$, $g' = -0.10$

$$\frac{-0.11(3.9-0.10t) \overset{+}{-} (-0.10)(2.1-0.11t)}{(3.9-0.10t)^2}$$

$$= \frac{-.429 + 0.011t + .2.1 - 0.011t}{(3.9-0.10t)^2} = \frac{-0.219}{(3.9-0.10t)^2}$$

$$\frac{d}{dt}\Big|_{t=8} = \frac{-0.219}{(3.9-0.10(8))^2} = -0.022789$$

$\frac{I}{P}$ = % Imp / Total Production